5.5 Integration By Substitution

Example ① $$\int \sin x \cos x \, dx = \int u \, du = \frac{u^2}{2} + C = \frac{\sin^2 x}{2} + C$$

$u = \sin x$

$du = \cos x \, dx$

Example ② $$\int x e^{x^2} dx = \int \frac{1}{2} e^u du = \frac{1}{2} e^u + C = \frac{1}{2} e^{x^2} + C$$

$u = x^2$

$du = 2x \, dx$

$\frac{1}{2} du = x \, dx$

Example ③ $$\int \sin 5x \, dx = \int \frac{1}{5} \sin u \, du = -\frac{1}{5} \cos u + C$$

$$= -\frac{1}{5} \cos 5x + C$$

$u = 5x$

$du = 5 \, dx$

$\frac{1}{5} du = dx$

Example ④ $$\int \sec \pi x \tan \pi x \, dx = \int \frac{1}{\pi} \sec u \tan u \, du = \frac{1}{\pi} \sec u + C$$

$$= \frac{1}{\pi} \sec \pi x + C$$

$u = \pi x$

$du = \pi \, dx$

$\frac{1}{\pi} du = dx$

Example ⑤ $$\int x \sin x^2 e^{\cos x^2} dx = \int -\frac{1}{2} e^u du = -\frac{1}{2} e^u + C$$

$$= -\frac{1}{2} e^{\cos x^2} + C$$

$u = \cos x^2$

$du = -2x \sin x^2 dx$

$-\frac{1}{2} du = x \sin x^2 dx$

or:

$u = e^{\cos x^2}$

$du = -2x \sin x^2 e^{\cos x^2} dx$

$-\frac{1}{2} du = x \sin x^2 e^{\cos x^2} dx$

$$= \int -\frac{1}{2} du$$

$$= -\frac{1}{2} u + C$$

$$= -\frac{1}{2} e^{\cos x^2} + C$$

Example ⑥ $\int x\sqrt{1-x}\,dx = -\int (1-u)\sqrt{u}\,du$

$u = 1-x$

$du = -dx$

$-du = dx$

$x = 1-u$

$= \int (u^{3/2} - u^{1/2})\,du$

$= \frac{2}{5}u^{5/2} - \frac{2}{3}u^{3/2} + C$

$= \frac{2}{5}(1-x)^{5/2} - \frac{2}{3}(1-x)^{3/2} + C$

Example ⑦ $\int \frac{1}{\sqrt{e^x - 1}}\,dx = \int \frac{1}{(u+1)\sqrt{u}}\,du$

$u = e^x - 1$

$du = e^x dx$

$= (u+1)dx$

$\frac{1}{u+1}du = dx$

$= \int \frac{2}{v^2+1}\,dv$

$= 2\tan^{-1} v + C$

$= 2\tan^{-1}\sqrt{u} + C$

$= 2\tan^{-1}\sqrt{e^x - 1} + C$

$v = \sqrt{u}$

$dv = \frac{1}{2\sqrt{u}}\,du$

$2\,dv = \frac{1}{\sqrt{u}}\,du$