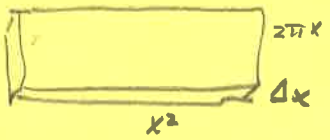
6.2 Volumes with Cylindrical Shells

Example ① Rotate the graph of $y = x^2$, $0 \le x \le 1$ around the y-axis. Find the volume.

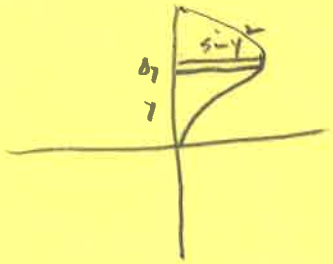
Flatten the shell:

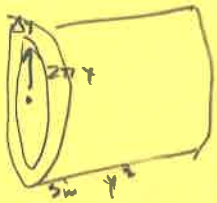


Volume of shell ≈ Volume of slab = $2\pi x \cdot x^2 \Delta x$

$$\text{Volume} = \int_0^1 2\pi x^3 \, dx = \frac{\pi x^4}{2}\Big|_0^1 = \pi/2$$

Example ② The graph of $x = \sin y^2$ $0 \le y \le \sqrt{\pi}$ is rotated around the x-axis. Find the volume.





Volume of shell ≈ $2\pi y \cdot \sin y^2 \, dy$

$$\text{Volume} = \int_0^{\sqrt{\pi}} 2\pi y \sin y^2 \, dy = \int_0^{\pi} \frac{2\pi}{2} \sin u \, du$$

$u = y^2$

$du = 2y\,dy$

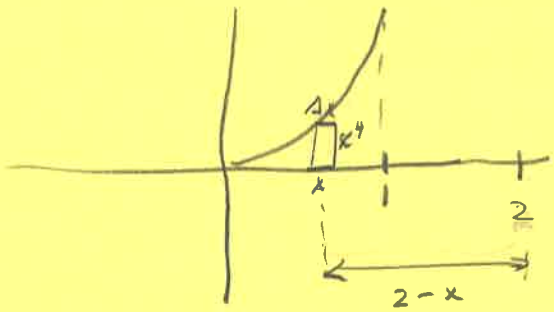
$\frac{1}{2} du = y\,dy$

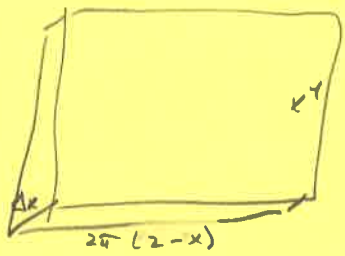
$$= -\pi \cos u \Big|_0^{\pi}$$

$$= -\pi(-1 - 1)$$

$$= 2\pi$$

Example (3) The graph of $y = x^4$ $0 \le x \le 1$ is rotated around $x = 2$. Find the volume.





Volume of shell $= 2\pi (2-x) x^4 \Delta x$

$$\text{Volume} = \int_0^1 2\pi(2-x)x^4\,dx = 2\pi\int_0^1 (2x^4 - x^5)\,dx$$

$$= 2\pi\left[\frac{2x^5}{5} - \frac{x^6}{6}\right]\Big|_0^1 = 2\pi\left(\frac{2}{5} - \frac{1}{6}\right)$$

$$= \frac{7\pi}{15}$$