(8.1) Integration By Parts

$$\frac{d}{dx}(fg) = f\frac{dg}{dx} + g\frac{df}{dx}$$

Let $u = f(x)$ $\quad v = g(x)$

$du = f'(x)\,dx$ $\quad dv = g'(x)\,dx$

$$fg = \int \frac{d}{dx}(fg)\,dx = \int f\frac{dg}{dx}\,dx + \int g\frac{df}{dx}\,dx$$

$$uv = \int u\,dv + \int v\,du$$

$$\boxed{\int u\,dv = uv - \int v\,du}$$ Integration By Parts Formula

Example ① $\int xe^x\,dx = xe^x - \int e^x\,dx = xe^x - x + C$

$u = x$ $\quad dv = e^x\,dx$

$du = dx$ $\quad v = e^x$

Example ② $\int x^2 \sin x\,dx = -x^2\cos x + \int 2x\cos x\,dx$

$u = x^2$ $\quad dv = \sin x\,dx$

$du = 2x\,dx$ $\quad v = -\cos x$

$= -x^2\cos x + 2x\sin x - 2\int \sin x\,dx$

$u = x$ $\quad dv = \cos x\,dx$

$du = dx$ $\quad v = \sin x$

$= -x^2\cos x + 2x\sin x + 2\cos x + C$

Example ③ $\int e^x \sin x\,dx = -e^x\cos x + \int \cos x\, e^x\,dx$

$u = e^x$ $\quad dv = \sin x\,dx$

$du = e^x dx$ $\quad v = -\cos x$

$= -e^x\cos x + e^x\sin x - \int e^x \sin x\,dx$

$u = e^x$ $\quad dv = \cos x\,dx$

$du = e^x dx$ $\quad v = \sin x$

solve for the integral

$$2\int e^x \sin x\,dx = -e^x\cos x + e^x\sin x$$

$$\int e^x \sin x\,dx = \frac{-e^x\cos x + e^x\sin x}{2} + C$$

Example 4 $\int x \ln x \, dx = \frac{x^2}{2}\ln x - \int \frac{x}{2}\,dx$

$u = \ln x \qquad dv = x\,dx$

$du = \frac{1}{x}dx \qquad v = \frac{x^2}{2}$

$= \frac{x^2}{2}\ln x - \frac{x^2}{4} + C$

Example 5 $\int \ln x \, dx = x\ln x - \int dx$

$u = \ln x \qquad dv = dx$

$du = \frac{1}{x}dx \qquad v = x$

$= x\ln x - x + C$

Example 6 $\int \sin^{-1} x \, dx = x\sin^{-1} x - \int \frac{x}{\sqrt{1-x^2}}\,dx$

$u = \sin^{-1} x \qquad dv = dx$

$du = \frac{1}{\sqrt{1-x^2}}dx \qquad v = x$

$= x\sin^{-1} x + \int \frac{1}{2\sqrt{u}}\,du$

$= x\sin^{-1} x + \frac{1}{2}\frac{u^{1/2}}{\frac{1}{2}} + C$

$= x\sin^{-1} x + \sqrt{1-x^2} + C$

$u = 1 - x^2$

$du = -2x\,dx$

$-\frac{1}{2}du = x\,dx$

Definite Integrals $\int_a^b u\,dv = uv\Big|_a^b - \int_a^b v\,du$

Example 7 $\int_0^1 x^2 e^x \, dx = x^2 e^x\Big|_0^1 - \int_0^1 2x e^x\,dx$

$u = x^2 \qquad dv = e^x dx$

$du = 2x\,dx \qquad v = e^x$

$= e - \left[2xe^x\Big|_0^1 - 2\int_0^1 e^x dx\right]$

$u = x \qquad dv = e^x dx$

$du = dx \qquad v = e^x$

$= e - 2e + 2e^x\Big|_0^1$

$= -e + 2e - 2$

$= e - 2$